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TRAJECTORIES OF CHARGED SOLID PARTICLES IN AN AIR JET UNDER THE INFLUENCE OF AN ELECTROSTATIC FIELD

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Summary

Among modern coating techniques electrostatic powder spraying is one of the most attractive. In this method the plastic particles are charged by a high voltage electrode inside a coating gun. The charged particles are entrained by an air jet and directed towards the object to be coated, which is electrically grounded.

In the present paper experimental trajectories of particles emerging from an electrostatic gun are determined. The equations of motion of particles in a jet are written in Lagrangean coordinates and solved numerically. Results are compared with the experimental data.

Introduction

The problem of motion of charged particles in an electric field has lately attracted attention in such diverse areas as electrostatic powder coating, dust collection and electrostatic precipitation. Osterle and Szasz [1] describe the electrostatic coating process and list the effects of different coating parameters on the process. White [2] and Robinson [3] review the whole field of electrostatic precipitation, with emphasis on the existing equipment and its application in various industries. They also summarize the existing theory of precipitation in terms of lumped parameter models. Soo [4] discusses the effect of electric charges on particle motion.

The present authors were interested in the motion of charged particles from the point of view of electrostatic powder coating. This technique can be described as follows. Plastic particles are transported by an air stream and fed into the coating gun. The gun consists of a long tube in which a high voltage electrode is positioned. The particles flow in the tube along the high voltage electrode, which imparts its charge to them. The jet of air and charged particles emerge from the barrel of the gun. The object to be coated is grounded; thus the emerged charged particles are attracted to it. The particles hitting the object form a coating layer, which is later fused together in an oven.

As part of the study of powder coating processes we were interested in the motion of a jet of charged particles in an electric field consisting of a point source (the gun tip) and a grounded flat plate. A previous work of the present authors [5] described the motion of solid particles in a turbulent air jet without the electric field. The present work takes us one step closer to the understanding of the real problem. This investigation is restricted to very dilute solid—air mixtures. Thus the effect of the particles on the total momentum of the jet as well as the interaction between the particles themselves are neglected. Experimental results are presented and these are then compared with theoretically obtained solutions.

Experimental investigation

The experiments were performed with the help of a SAMES electrostatic coating instrument. This instrument consists of a 90 kV d.c. power generator, a plastic-powder feeding mechanism, and a coating gun (Fig. 1).

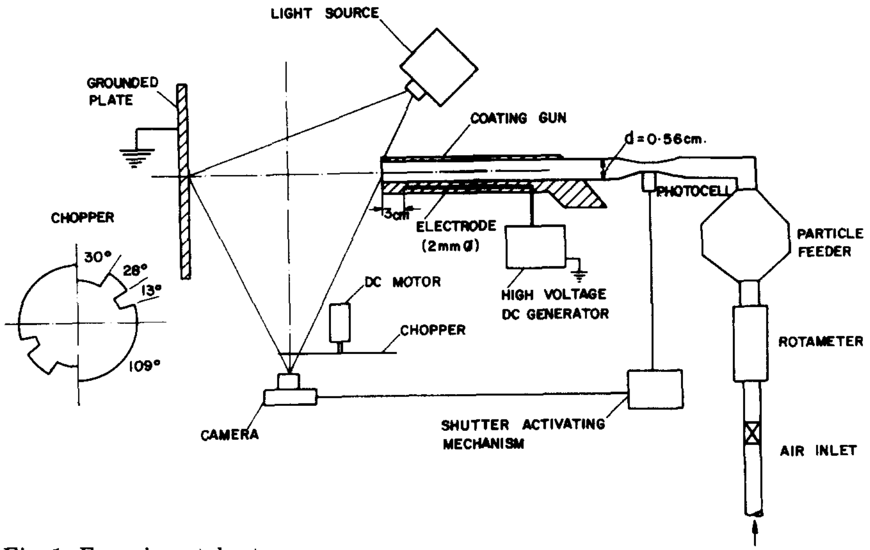

Fig. 1. Experimental set-up.

Polyethylene particles of 250—350 μm diameter were used in the experiments. They were fed by means of an air stream into the gun where they were charged by means of the high voltage electrode. The particles and the air emerged from the gun as a two-phase jet, directed at a grounded target to which they adhered.

We were interested in determining the trajectories, and velocities of the flying particles. In order to record the particle pathlines a photographic technique similar to the one described previously [5] was used. The jet was illuminated by means of a projector equipped with a 1-mm slit. The projector was positioned vertically in such a way that the light passing through the slit illuminated a vertical plane formed by the gun axis and the direction of gravity. The illuminated region was 1—5 mm thick. The lighted particles were photographed by means of a 35-mm camera. Velocities and pathlines were obtained by chopping the light reaching the camera by means of a slotted disc run by an adjustable speed d.c. motor.

The experiments were designed in such a way as to show clearly the effect of the electric field on the trajectories of the particles. In our previous work, performed without the electric field but with the same particles, it was noted that at air velocities lower than 500 cm/sec the effect of gravity on the particles became more pronounced, resulting in the disengagement of the particles from the air jet. In the present study we have used an air velocity of 380 cm/sec in anticipation that the presence of the electric field will prevent disengagement. The grounded plate was placed at a distance of 16 cm from the gun tip, while experimental data were obtained up to a distance "x" of 12 cm.

Figure 2 provides a collection of pathlines obtained from the experiments.

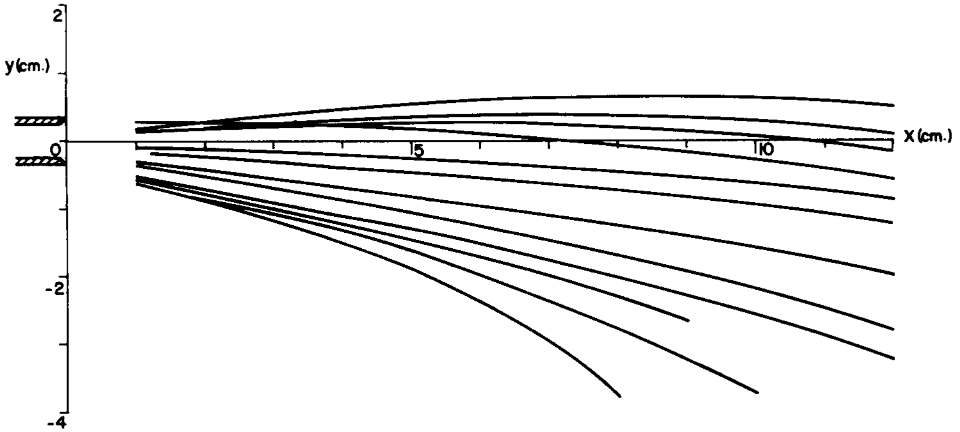

Fig. 2. Experimental particle pathlines for an air velocity of 380 cm/sec at gun exit.

It is seen that particles emerging from the gun at angles close to zero lose very little of their original altitude, whereas particles emerging at negative angles of more than 10 degrees seem to be affected more by gravity. We were surprised to note that there were no particles emerging from the gun at angles higher than +10 degrees. This is in sharp contrast to results previously obtained without the electric field. Several of the lower pathlines seem not to emerge from within the nozzle. Although interparticle or particle—wall collisions at the nozzle exit could account for this occurrence we feel that a more satisfactory explanation is needed. In Fig. 3 we present the velocity vectors at two distances from the exit together with the axial velocity distribution of the free air jet. Here one notes that a large portion of the particles

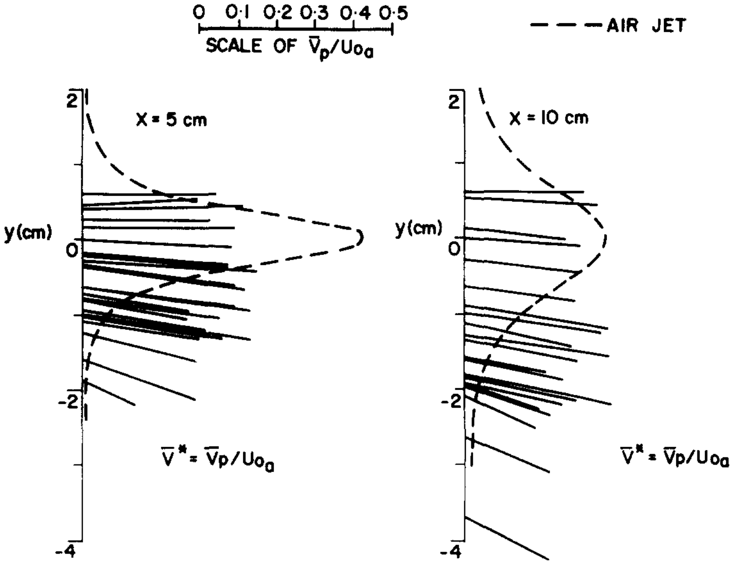

Fig. 3. Experimental particle velocity vectors in an electrostatic field and air jet axial velocity distributions.

at a given cross-section are at a uniform velocity. Only the particles at the lower end have lower velocities and are directed at higher negative angles.

Theory

An analytical treatment is now presented for the calculation of trajectories and velocities of charged particles within a decelerating turbulent air stream, under the influence of gravity and an electric field. Since very dilute solid—air mixtures are considered, the effect of the particles on the air jet velocities and of particle—particle interaction may be ignored. Hence, the trajectory and velocity variation of a single particle in a known air jet can be calculated. The particle is taken to be located in the plane defined by the axis of a round jet intersecting with the direction of gravity (Fig. 4). This configuration corresponds to the one realized experimentally by illuminating a vertical plane of the jet axis.

The equations of particle motion for this case are written in Lagrangean coordinates as follows:

$$m_p \frac{d^2 x}{dt^2} = F_x + qE_x \tag{1}$$

$$m_p \frac{d^2 y}{dt^2} = F_y - m_p g \tag{2}$$

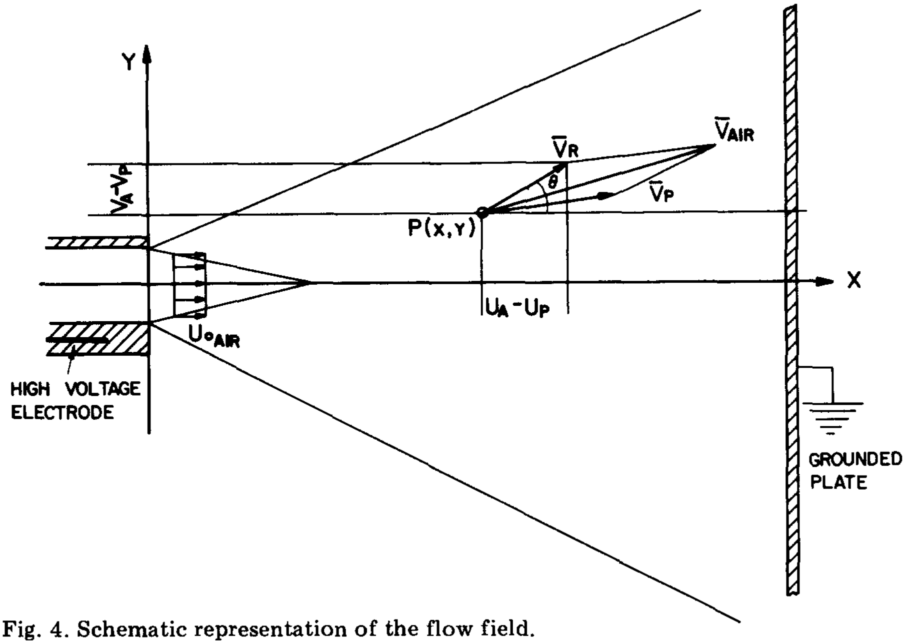

Fig. 4. Schematic representation of the flow field.

where m_p is the mass of the particle, F_x and F_y are, respectively, the x and y components of the drag force acting on the particle due to the difference between the air and particle velocities, q is the particle charge, and E_x is the field strength in the x direction at the given point. In eqn. (2) it was assumed that the y component of the force on the particle due to the field E is negligible.

Expressing the drag force in terms of the drag coefficient we arrive at the following equations of particle motion:

$$\frac{d^2x}{dt^2} = \frac{3}{4}\,\frac{C_D}{(\cos\theta)D_p}\cdot\frac{\rho_a}{\rho_p}\left|U_a - \frac{dx}{dt}\right|\cdot\left(U_a - \frac{dx}{dt}\right) + \frac{6qE_x}{\pi D_p^3\rho_p} \tag{3}$$

$$\frac{d^2y}{dt^2} = \frac{3}{4}\,\frac{C_D}{(\sin\theta)D_p}\cdot\frac{\rho_a}{\rho_p}\left|V_a - \frac{dy}{dt}\right|\cdot\left(V_a - \frac{dy}{dt}\right) - g \tag{4}$$

where U_a and V_a are, respectively, the x and y components of the air jet velocity at the point of consideration, θ is the angle between the relative velocity and the x direction. The drag coefficient is given by the expression derived by Gilbert *et al.* [6]:

$$C_D = \frac{24}{\mathrm{Re}}\left[\frac{28\mathrm{Re}^{-0.5} + 0.48}{24\mathrm{Re}^{-1.0}}\right] \tag{5}$$

Here the Reynolds number is defined as:

$$\mathrm{Re} = \frac{\rho_a V_R D_p}{\mu_a} \tag{6}$$

V_R being the relative velocity between the air and the particle, μ_a the air viscosity, and ρ_a the air density.

The velocity components U_a and V_a of the air jet were determined by using the potential core and mixing region concepts when close to the exit of the jet and Tollmien's solution of a circular turbulent jet for greater distances [7] (Fig. 4).

The charge q acquired by the particle while flowing along the high voltage electrode can be due to ion bombardment, ion diffusion, or a combination of both. For particles larger than 10 microns in diameter such as in the case of plastic powder coating, the ion bombardment mechanism is predominant [1]. Assuming that the time of particle flight inside the charging region is long enough for the acquisition of the final or limiting charge, then the following expression for q may be used [1, 2, 3]:

$$q = \frac{3\epsilon_p}{\epsilon_p + 2} E_0 \cdot \frac{D_p^2}{4} \qquad (7)$$

where ϵ_p is the relative dielectric constant of the particles and E_0 the charging field strength. The value of ϵ_p for polyethylene is given in the literature [8] as 2.3.

The electric field accelerating the particles from the gun to the object is formed due to the interaction between the tip of the electrode inside the gun, which is at 90 kV, the grounded plate and the charged particles in the field. The differential equation of this field is:

$$\nabla^2 V = \frac{n}{\epsilon} \qquad (8)$$

where V is the voltage, n is the charge density of the particles, and ϵ the permittivity of the field.

The potential field which enters into the equation of motion is related to the voltage by:

$$E = -\nabla V \qquad (9)$$

In our case, the number of charged particles in the field is small and the effect of their charge on the field may be neglected. Therefore, eqn. (8) may be simplified to:

$$\nabla^2 V = 0 \qquad (10)$$

which is the Laplace equation.

The solution of this equation for a point source located at a given distance from a rectangular flat plate is quite difficult, as the problem is three-dimensional. We were not interested in very accurate values of voltage in the whole field but rather in values of the potential. Our data were taken in the vertical plane passing through the point source and perpendicular to the plate. Thus we were interested in the values of the voltage potential in this plane only. Although the exact solution is not known, it can be bounded by upper and lower limits provided by two available solutions from the literature. The first one is the solution of a point source located at a distance "a" from an infinite grounded plate. This solution, which gives a higher voltage drop, hence a stronger force, than the one for a finite plate, is:

$$V = \text{const}\left[\frac{1}{[x^2 + y^2]^{1/2}} - \frac{1}{[(2a - x)^2 + y^2]^{1/2}}\right] \tag{11}$$

and

$$E_x = -\frac{\partial V}{\partial x} = \text{const}\left[\frac{x}{[x^2 + y^2]^{3/2}} + \frac{2a - x}{[(2a - x)^2 + y^2]^{3/2}}\right] \tag{12}$$

The second solution resulting in a weaker force is that of a point source located at an infinite distance from a grounded plate. This solution is:

$$V = \text{const}\left[\frac{1}{[x^2 + y^2]^{1/2}}\right] \tag{13}$$

and

$$E_x = -\frac{\partial V}{\partial x} = \text{const}\left[\frac{x}{[x^2 + y^2]^{3/2}}\right] \tag{14}$$

In eqns. (11)—(14) the distance "x" is measured from the electrode tip. Equation (14) was used in eqn. (3) for computational purposes. Thus, the electrostatic attraction was accounted for rather conservatively.

The two differential eqns. (3) and (4) were solved numerically on an IBM 370/165 digital computer by using the Runge—Kutta method. The constant in eqn. (14) was determined from the geometry of the coating gun and the data of 90 kV potential at the electrode tip as supplied by the manufacturer of the instrument.

The numerical solutions were initiated at a given point which was specified by its location and the air and particle velocity vectors, and proceeded thereafter.

Comparison between theory and experiment

Figure 5 presents typical calculated particle trajectories together with their experimental counterparts. As seen, some of the computed lines follow the experimental data quite closely, whereas the other ones show marked deviations. There are several reasons for these discrepancies, such as:

(1) The deviation of particle size from the assumed 300 μm diameter used in the computations.

(2) The expression used for the drag coefficient, eqn. (5), is the least applicable for the transition region of Re = 2 ÷ 100.

(3) The deviation of the actual electrical potential at the electrode tip from the one supplied by the manufacturer as 90 kV.

In order to keep matters in the right perspective we lift from Fig. 5 two of the less agreeable lines, 1 and 7, and replot them in Fig. 6 together with the trajectories obtained under identical conditions but without an electric field. The effect of the additional electrical force is clearly apparent, especially at the higher negative angles. Figure 7 presents particle velocities as a function of the axial distance from the jet exit, for the case of particles emerging from the center of the nozzle at angles close to 0°. The particles come out of the nozzle at a velocity which is about a third of that of the air. This is similar to the results obtained previously without the electric field. We have also plotted on Fig. 7, a computed line corresponding to $V^* = 0.35$ at $x = 2$ cm. We note

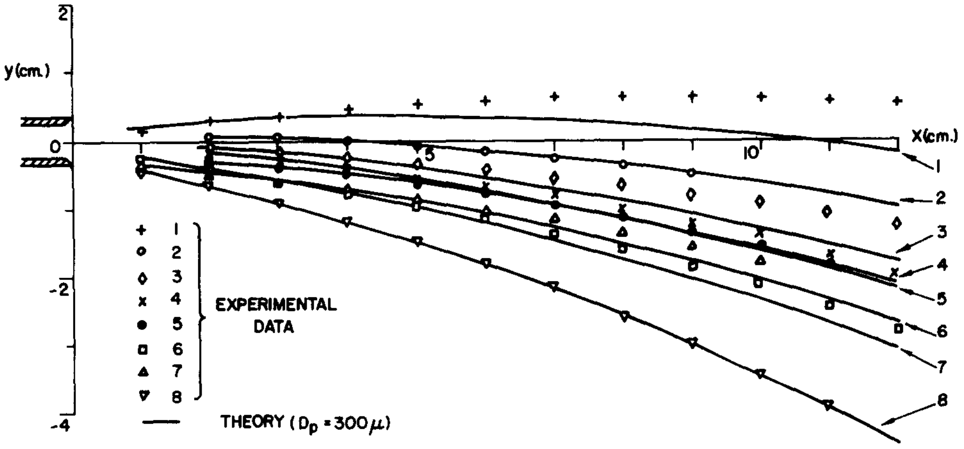

Fig. 5. Comparison between experimental and theoretical pathlines.

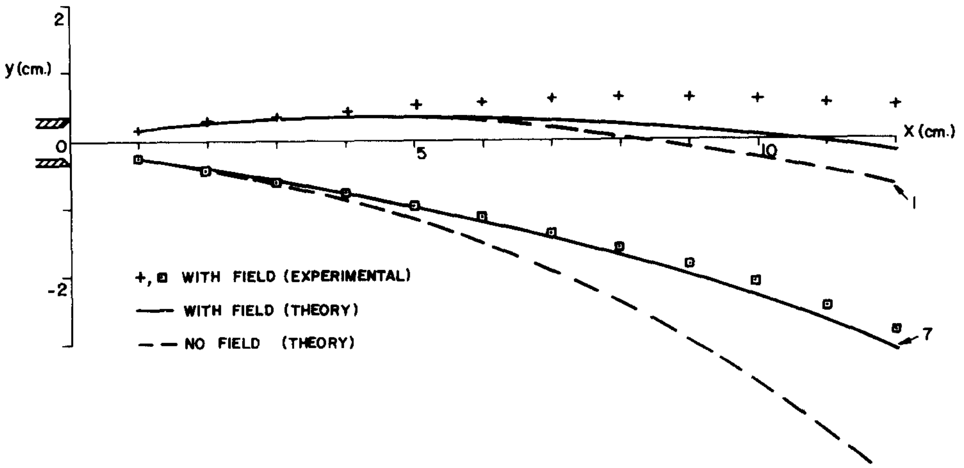

Fig. 6. Comparison between particle pathlines with and without the electric field.

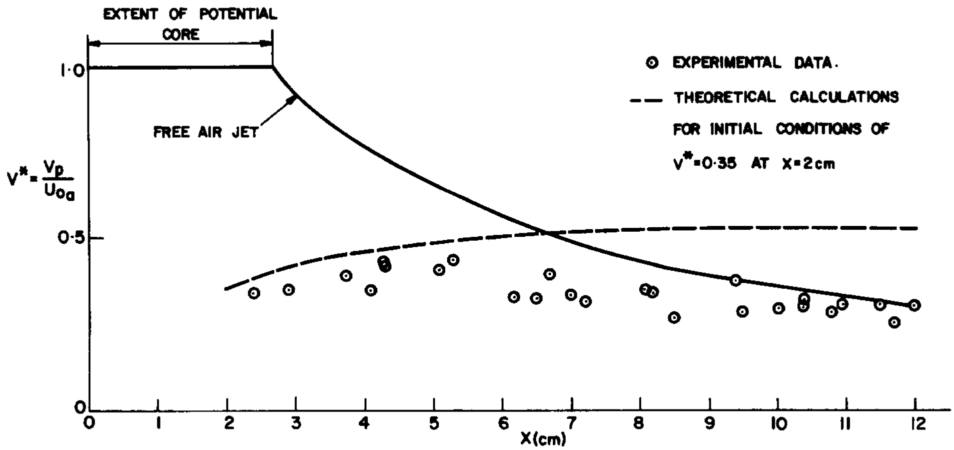

Fig. 7. Dimensionless velocities of particles emerging horizontally from the nozzle center together with the centerline air-jet velocity.

that the computed line shows a more pronounced increase of velocity with the distance than the experimental results. We believe that this is due to an assumed drag coefficient which is higher than the one encountered in reality. The particle Reynolds number in our experiments is in the range 20—40, where existing data on drag coefficients vary appreciably [4].

Summary and Conclusions

This paper brings us one step further in understanding the process of electrostatic powder coating.

Experimental trajectories of particles flowing in an air jet and under the influence of an electric field were obtained. It was observed that the influence of the electric field results in a more uniform flow of the particles when compared with the previously investigated similar case without the electric field, a result which is of important practical significance.

It was also observed that the presence of the electric field maintains the particles within the jet, under air jet conditions where, without the electric field, a disengagement of particles from the air stream occurred.

The Lagrangean equations of motion, with the addition of the electrical force, were solved numerically for the vertical plane through the jet axis and the results compared to the corresponding experimental findings. The marked effect of the electric field on the trajectories of particles was observed both theoretically and experimentally.

The occasional deviations between theory and experiments which were observed may be explained by uncertainties in assessing the drag coefficient, particle size and shape, and electric field intensity.

The results of the present study are by no means limited to electrostatic powder coating. As an example one may mention the vast field of electrostatic precipitation where similar methods of analysis may be applied.

Acknowledgements

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Nomenclature

a	distance between the point source and the grounded plate
C_D	drag coefficient
D_{jet}	nozzle exit diameter
D_p	diameter of particle
E	electric field
E_0	charging field
$\bar{F}$	total drag force
F_x	axial component of the drag force
F_y	transverse component of the drag force
g	acceleration of gravity
m_p	mass of the particle
n	space charge density
q	limiting charge of the particle
Re	Reynolds number — $\rho_a V_R D_p / \mu_a$
t	time
U_a	axial air jet velocity
U_{0a}	air velocity at exit of gun
U_p	axial particle velocity
V	voltage
V_a	transverse air jet velocity
V_p	transverse particle velocity
$\bar{V}_p$	particle velocity
$\bar{V}_R$	relative velocity between the air and the particle
$\bar{V}^*$	dimensionless velocity — $\bar{V}_p / U_{0a}$
x	axial coordinate
y	transverse coordinate

Greek symbols

ρ_a	air density
ρ_p	density of the particle
μ_a	air viscosity

θ angle between relative velocity and x direction
ϵ_p relative dielectric constant of the particle
ϵ permittivity

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